HW 2: 2.2 Separable eqns, slope fields

2.2: Separable Diff. Eq. continued

Entry Task: A spherical snowball melts (changes volume) at a rate proportional to its surface area.
Initial the volume is 1000 in^3.
Two min. later the volume is 729 in^3.

Solve the differential equation below (see next page for explanation of equation) and use the initial conditions, then predict how long it will take for the snowball to completely melt.

$$\frac{dV}{dt} = -DV^{\frac{2}{3}}$$

Set-up Notes: Note volume, radius and surface area are all changing as functions of time. Let's denote

$$V = V(t), r = r(t), S = S(t)$$

And recall: $V = \frac{4}{3}\pi r^3, S = 4\pi r^2$

And so $r = \left(\frac{3}{4\pi}\right)^{\frac{1}{3}} V^{\frac{1}{3}}$.

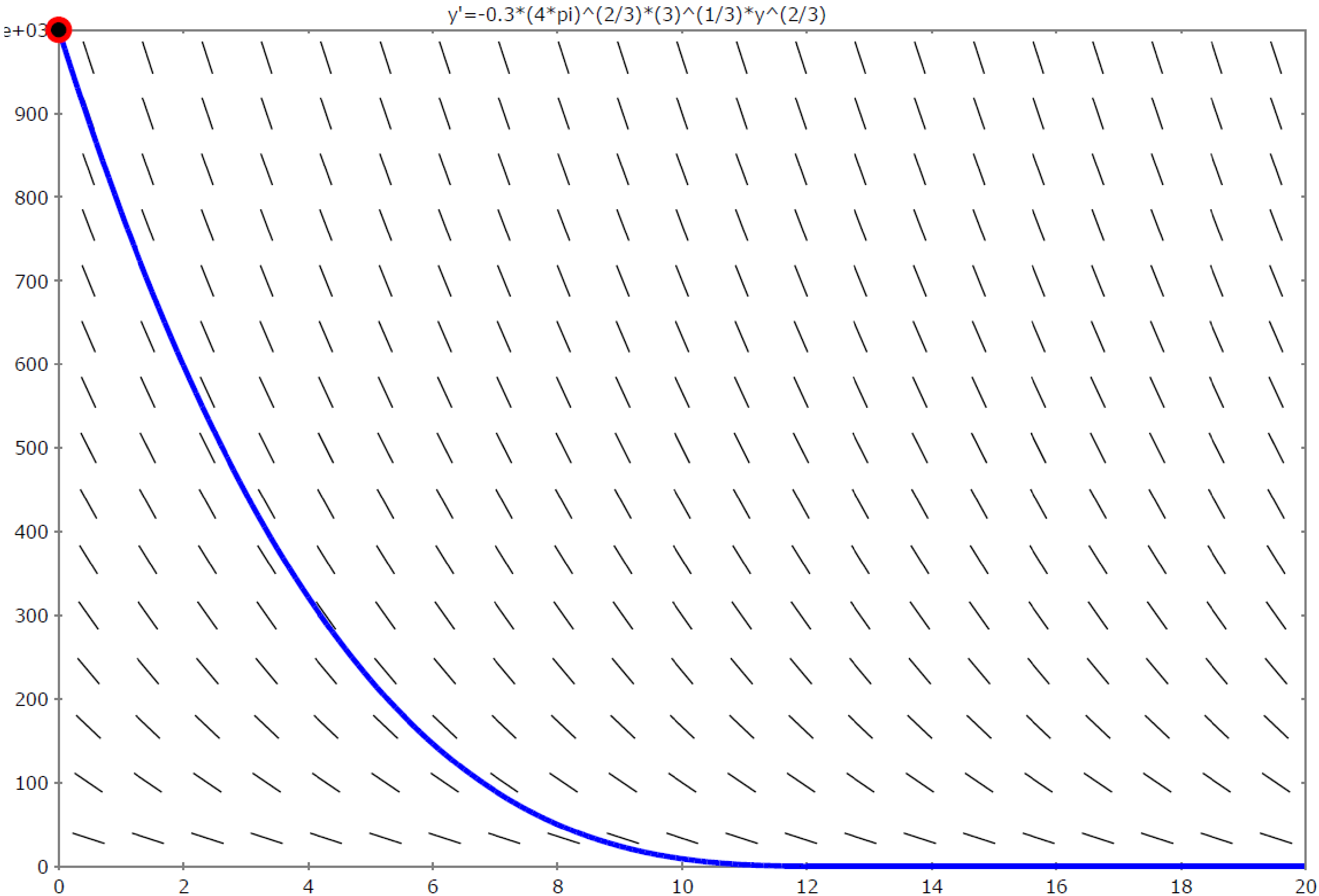
From the given assumption:

$$\frac{dV}{dt} = -k4\pi r^2 = -k4\pi \left(\frac{3}{4\pi}\right)^{\frac{2}{3}} V^{\frac{2}{3}}$$

Let

$$D = k4\pi \left(\frac{3}{4\pi}\right)^{\frac{2}{3}} = k(4\pi)^{\frac{1}{3}} 3^{\frac{2}{3}}$$

Slope field for the Snowball problem

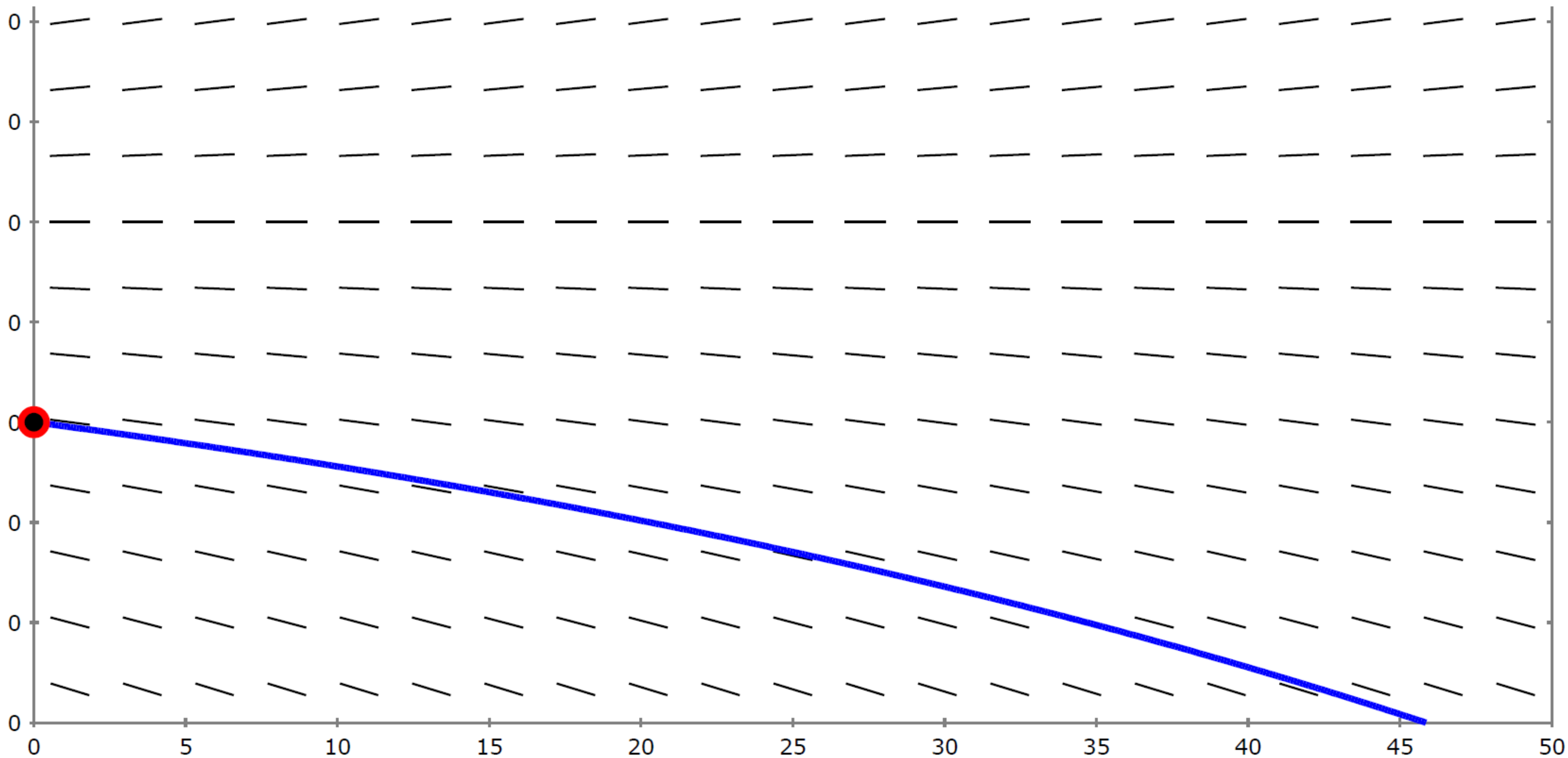


Example:
You have \$30,000 in a bank account.
- The account earns 2% annual interest, compounded continuously.
- In addition, you withdraw money throughout the year totaling about \$1000/year.

When will you run out of money?

How much do you need in the account initially so you never run out of money?

Slope field for Bank Account Example



Example:
Consider
$$\frac{dy}{dx} = 3x - y$$
This is NOT separable. It is "linear" and we will discuss a method next time for this type.

But if you leave this course, you may encounter a method called "change of variable" to "fix" a problem like this. Let's try one.

Assume I tell you to let $v = 3x - y$
Find
$$\frac{dv}{dx} =$$

This new equation is separable!! Solve it, then rewrite your final answer in terms of y and x.

Slope Field for last example